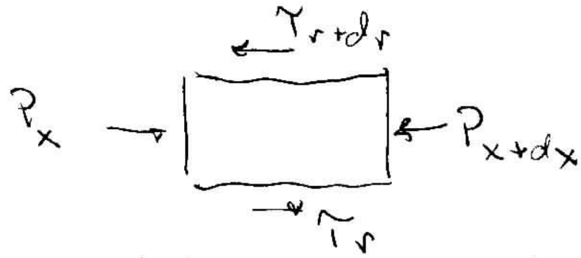
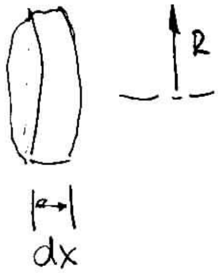
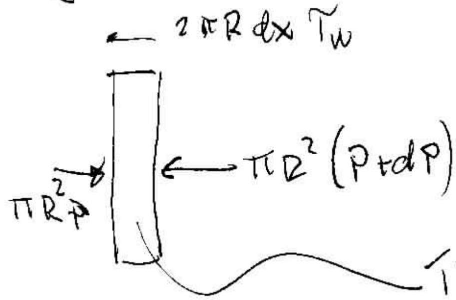
Laminar flow in ○ tubes...



Do force balance on ○ ring ...
or
wafer





This leads to
$$\frac{dP}{dx} = -\frac{2}{R}\tau_w$$
(used below)

both lead to

$$\underbrace{\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right)}_{r \text{ variation}} = \underbrace{\frac{dP}{dx}}_{x \text{ variation}}$$

LHS   RHS

Indep of each other

Conclude that both sides = the same (maybe) unknown ¢

So $\dfrac{dP}{dx} = ¢ = -\dfrac{2}{R}\tau_w$

In which case LHS becomes after $\int$ twice

$$u(r) = \frac{1}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

B.C. $u\big|_{r=R} = 0$ (no slip)

$\dfrac{\partial u}{\partial r}\Big|_{r=0} = 0$ (¢ sym)

so

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Fully dev. parab. flow!

Still have the pesky dP/dx, but...

$$\bar{V} = \frac{2}{R^2}\int_{r=0}^{R} u(r)\,dr = \cdots = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

use this to eliminate the dP/dx term

so

$$u(r) = 2\bar{V}\left(1-\frac{r^2}{R^2}\right)$$

↑ easy to measure!

Note that

$$u\Big|_{r=0} = \boxed{U_{max} = 2\bar{U}}$$

ON to pressure drop

RHS

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L} = \text{¢} \quad \text{from } \underbrace{x_1}_{P_1} \text{ to } \underbrace{x_2}_{P_2} = x_1 + L$$

Thus for $\frac{dP}{dx}$ from $u(r)$ soln

$$\Delta P = P_1 - P_2 = \frac{32\mu L\bar{U}}{D^2} \quad (*)$$

↑ Pressure Drop ΔP_L

Due to friction in fluid flow ~ related to dissipation term.

$\mu \to 0$ No friction loss, no ΔP_L

Common to use

$$\Delta P_L = f\,\frac{L}{D}\,\frac{\rho\bar{U}^2}{2} \quad (*)$$

f ~ Darcy friction factor

$$\text{so } f = \frac{8\tau_w}{\rho\bar{U}^2}$$

Also called Darcy-Weisbach friction factor.

Also recall Fanning friction factor

$$C_f = \frac{2\tau_w}{\rho\bar{U}^2} = f/4$$

Set both (*) to eachother to get

Circ. tube, laminar

$$f = \frac{64\mu}{\rho D\bar{U}} = \frac{64}{Re}$$

Recall head loss $h_L = \dfrac{\Delta P_L}{\rho g} = f \dfrac{L}{D} \dfrac{\bar{U}^2}{2g}$ Recall $\Delta p = \rho g h$

laminar flow only

Additional height of fluid to overcome frictional losses in pipe flow.

Required pump power $\dot{W}_{pump,L} = \dot{V}\,\Delta P_L = \dot{V} \rho g h_L = \dot{m} g h_L$

where $\dot{V}$ is avg volume flow rate

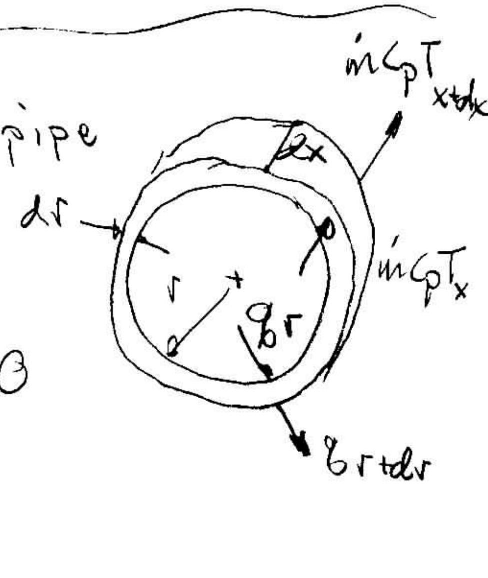
But
For a horizontal tube $\bar{U} = \dfrac{(P_1 - P_2)R^2}{8\mu L} = \dfrac{\Delta P\, D^2}{32\mu L}$

in which case

$$\dot{V} = \bar{U} A_c = \frac{(P_1 - P_2)R^2}{8\mu L} \cdot \pi R^2 = \frac{\Delta P \pi D^4}{128 \mu L}$$ Poiseuille's Law

So ↓ pump power by 16x by doubling the diameter.

On to heat transfer ...
Look at an annular section of flow down a pipe



Ⓔ balance (S.S.)

$$\dot{m} c_p T\big|_x - \dot{m} c_p T\big|_{x+\Delta x} + q\big|_r - q\big|_{r+\Delta r} = 0$$

recall $\dot{m} = \rho U A_c = \rho U 2\pi r\, dr$

leads to

$$u \frac{\partial T}{\partial x} = \frac{-1}{2\rho c_p \pi r\, dx} \frac{\partial q}{\partial r}$$

But Fourier's Law in radial dir $\dfrac{\partial q}{\partial r} = \dfrac{\partial}{\partial r}\left(-2\pi k\, dx \dfrac{\partial T}{\partial r}\right) = -2\pi k\, dx \dfrac{\partial}{\partial r}\left(r \dfrac{\partial T}{\partial r}\right)$

so finally
$$\boxed{u\frac{\partial T}{\partial x} = \frac{\alpha}{r}\frac{\partial}{\partial r}\left(r\frac{\partial T}{\partial r}\right)}$$

Wait this is just cyl.
$$\rho C_p \frac{DT}{Dt} = k\nabla^2 T + q'''_{gen}$$
Could have just saved 2 pages of work!

Solution for <u>ϕ</u> <u>heat</u> <u>flux</u> <u>surface</u>

Recall for fully dev. flow in ○ pipe with ϕ surf. heat flux...

$$\frac{\partial T}{\partial x} = \frac{dT_s}{dx} = \frac{d\bar{T}}{dx} = \frac{2q_s}{\rho \bar{U} c_p R} = \phi$$

↑ put this in above eqn. and use our velocity profile to get

$$\frac{4q_s}{kR}\left(1-\frac{r^2}{R^2}\right) = \frac{1}{r}\frac{d}{dr}\left(r\frac{dT}{dr}\right) \qquad \text{just an ODE!}$$

$\int\int$ twice ...

$$T(r) = \frac{q_s}{kR}\left(r^2 - \frac{r^4}{4R^2}\right) + c_1 \ln r + c_2$$

B.C. @ $r=R$ $T=T_s$

@ $r=0$ $\partial T/\partial r = 0$ (rad. sym.) to get c_1 + c_2

$$T(r) = T_s - \frac{q_s R}{k}\left(\frac{3}{4} - \frac{r^2}{R^2} + \frac{1}{4}\frac{r^4}{R^4}\right)$$

can then calc mean T...

$$\bar{T} = T_s - \frac{11}{24}\frac{q_s R}{k}$$

Combine to get

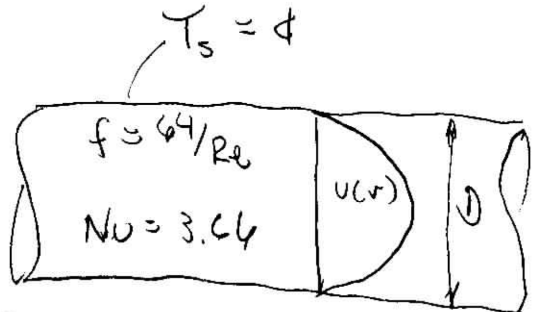
$$h = \frac{24}{11}\frac{k}{R} = \frac{48}{\pi}\frac{k}{D} \doteq 4.36\frac{k}{D}$$

so ∴

$$Nu = \frac{hD}{k} = 4.36 \quad \left\{ \begin{array}{l} \text{Fully Dev} \\ \bigcirc \text{ tube} \\ \text{¢ } q \text{ flux at surface} \end{array} \right.$$

Duplicate above process for ¢ T_s at surface

$$Nu = 3.66$$



For non ◯ tubes ... charts of

	T_s ¢	q_s ¢	f
◯	Nu = 3.66	4.36	64/Re
▭	–	–	.../Re
⬭	–	–	.../Re
△	–	–	.../Re

What about entry region stuff

O, Laminar Entry

$$Nu = 3.66 + \frac{0.065\left(\frac{D}{L}\right) Re\,Pr}{1 + 0.04\left[\left(\frac{D}{L}\right) Re\,Pr\right]^{2/3}}$$

O, Laminar but large T_s + T_{fluid} difference Entry

$$Nu = 1.86\left(\frac{Re\,Pr\,D}{L}\right)^{1/3}\left(\frac{\mu_b}{\mu_s}\right)^{0.14}$$

plates, Laminar ENTRY

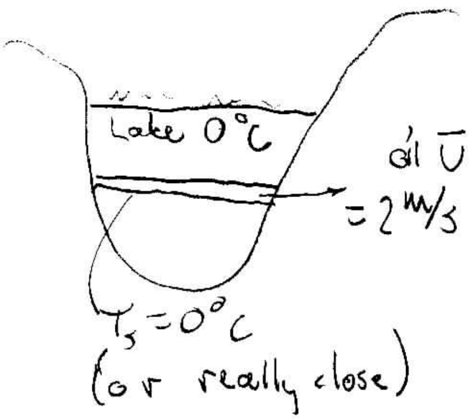
$D_h = 2 \cdot$ plate spacing

for $Re \leq 2800$

$$Nu = 7.54 + \frac{0.03\left(\frac{D_h}{L}\right) Re\,Pr}{1 + 0.016\left[\left(\frac{D_h}{L}\right) Re\,Pr\right]^{2/3}}$$

And so on...

Ex| Oil pipeline in a lake



Pipe $D = 0.3\,m$

$T_{inlet} = 20°C$

Pipe $L = 200\,m$

Calc: T_{exit} for oil

- heat trans. rate
- Pump. requirements

Assume:

- S.S.
- $T_s \approx 0°C$
- Neg. resist. in pipe
- smooth pipe walls
- flow is full. dev. in pipe section

We dont know T_{exit} so can't calc T_{bulk} so just try evaluating prop. @ $T_{inlet} = 20°C$.

For oil @ 20°C

$\rho = 888.1 \frac{kg}{m^3}$ $\quad \nu = 9.429 \times 10^{-4}\ m^2/s$

$C_p = 1880\ J/kg \cdot K$

$k = 0.145 \frac{W}{mK}$

$Pr = 10,863$

Here we go...

$$Re = \frac{\bar{U} D}{\nu} = 636 \underset{\text{crit } Re}{< 2300} \Rightarrow \text{Assume lam. flow}$$

$$L_{therm\ entry\ length} \approx 0.05\, Re\, Pr\, D = 103,600\ m \ll \text{pipe length}$$

So have thermally developing flow hence

$$Nu = \frac{hD}{k} = 3.66 + \frac{0.065 \left(\frac{D}{L}\right) Re\, Pr}{\left[1 + 0.04\left[\left(\frac{D}{L}\right) Re\, Pr\right]^{2/3}\right]} = 33.7$$

hence

$$h = Nu \frac{k}{D} = 16.3 \frac{W}{m^2 K}$$

But we also know $A_s = \pi D L = 188.5\ m^2$

$\dot{m} = \rho A_c \bar{U} = 125.6\ kg/s$

So exit temp is

$$T_e = T_s - (T_s - T_i) \exp\left(-\frac{h A_s}{\dot{m} c_p}\right) = 19.74°C$$

Not much of a T drop. This would give a bulk $T_b = 19.87°C$.

We did ok ... no need to recalculate.

Calc $\Delta T_{lm} = \frac{(T_i - T_e)}{\ln\left(\frac{T_s - T_e}{T_s - T_i}\right)} = -19.87\,^\circ C$

so $\dot{q} = h A_s \Delta T_{lm} = -6.11 \times 10^4$ W

Lose heat 61.1 kW flowing in pipe.

Our laminar flow is hydrodynamically fully developed (unlike thermal which was not)

thus $f = \frac{64}{Re} = 0.1006$

so pressure drop is

$$\Delta P = f\left(\frac{L}{D}\right)\frac{\rho \bar{U}^2}{2} = \ldots = 1.19 \times 10^5 \ N/m^2$$

and so $\dot{W}_{pump} = \frac{\dot{m}\,\Delta P}{\rho} = 16.8$ kW (just to overcome friction in the flow.)

Turb. flow in tubes

- Turb. flow is often used 'cause of increased $\dot{q}$ transfer
- If $Re > 10{,}000 \Rightarrow$ fully turb.
- Lots of correlations here. See text. Follow directions on the package.

Transitional flow region... same situation... lots of correlations

Rough pipes same

⋮
